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PHYSICS RESEARCH ARTICLE

Effect of reservoir crowding on a two species asymmetric exclusion process

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Abstract: We study the effect of reservoir crowding in the non-equilibrium steady states of a two species asymmetric exclusion process coupled to a reservoir of finite resources. Here the entry and exit rate of both the species depend on the availability and storage capacity of particles in the reservoir. Depending on the values of the parameters the system can be in different phases.

Keywords: TASEP; Two species; Reservoir

1. Introduction

The Totally Asymmetric Simple Exclusion Process (TASEP) is a cornerstone model in non-equilibrium statistical mechanics, widely used to study driven diffusive systems. It describes particles moving unidirectionally on a one-dimensional lattice, with the exclusion principle ensuring that no two particles occupy the same site simultaneously. This simple rule leads to rich collective behaviour, including phase transitions, shock waves, and density fluctuations. TASEP has been instrumental in understanding real-world systems such as traffic flow, where shock waves correspond to traffic jams, and biological processes like molecular motor movement and protein synthesis, where particles compete for limited space [1-3].

A significant extension of TASEP investigates the reservoir crowding effect, where particles enter and exit the lattice based on the population of a finite reservoir. This setup models resource constraints, common in biological, ecological, and industrial systems. The dynamic coupling of entry and exit rates to the reservoir creates feedback mechanisms that regulate particle flow, leading to novel steady-state behaviours such as phase transitions and non-trivial density profiles. This model highlights how finite resources and feedback loops influence system dynamics, offering insights into processes like molecular transport in cells or supply chain management [4].

Another important variant is the two-species TASEP, which studies oppositely charged particles moving under an electric field. This model demonstrates spontaneous symmetry breaking, where the currents of the two species become unequal despite symmetric conditions. Such behaviour arises from subtle interactions and boundary effects, providing a framework to study phenomena like ion transport in biological channels or electrochemical cells. The model's ability to capture macroscopic asymmetries from microscopic rules makes it a powerful tool for exploring non-equilibrium systems [5].

Further, a minimalistic model of driven diffusive systems in one dimension incorporates short-range interactions and unbounded noise to study spontaneous symmetry breaking. This model challenges the conventional understanding that symmetry breaking is suppressed in low dimensions due to fluctuations. Instead, it shows how driven motion and noise can stabilize asymmetric states, even without long-range interactions. This addresses a long-standing question in statistical mechanics and opens new avenues for studying pattern formation, biological processes, and material science [6].

Here we have investigated a closed-system TASEP with two particle species and a reservoir explores constrained environments where the total particle number is fixed. The reservoir dynamically regulates particle entry and exit, creating a feedback loop that influences steady states. This model provides insights into resource competition and collective behaviour, with applications in biophysics (e.g., ribosomes on mRNA) and traffic engineering (e.g., vehicles on roads). By combining analytical techniques like mean-field approximations with numerical simulations, it offers a comprehensive understanding of how microscopic interactions lead to macroscopic phenomena in constrained systems.

In summary, these TASEP variants-ranging from reservoir-coupled models to two-species systems and minimalistic symmetry-breaking frameworks-advance our understanding of non-equilibrium systems. They provide versatile tools to study phase transitions, symmetry breaking, and collective behaviour in driven and constrained environments, with broad applications across physics, biology, and engineering.

2. Requisite Mathematical Tool (TASEP)

The Totally Asymmetric Exclusion Process (TASEP) is a fundamental nonequilibrium statistical mechanics model describing unidirectional particle movement on a lattice with no backward motion or overtaking. Under open boundary conditions, it has been exactly solved in one dimension, making it essential for studying transport far from equilibrium [1].

2.1. TASEP Rules

- Each site is either occupied or empty (exclusion principle).
- Particles move rightward with probability p, only if the next site is empty.
- Two setups: **Open TASEP** (particle injection/removal at boundaries) and **Periodic TASEP** (closed-loop motion).

2.2. Variants of TASEP

- **Open TASEP:** Particles enter at rate α, exit at rate β (models ribosome translation).
- **Periodic TASEP:** Conserves particle number (used in circular molecular motors).
- **TASEP with Langmuir Kinetics:** Particles attach/detach randomly (models DNA transcription).
- **Multi-Species TASEP:** Different particle types follow distinct rules (traffic and biological transport).

2.3. *Statistical properties*

TASEP exhibits interesting statistical properties, including:

2.3.1. Phase Transitions

- In **open TASEP**, different phases arise depending on boundary rates α and β:
  - **Low-density phase** ($\rho<1/2$): Determined by the entry rate α.
  - **High-density phase** ($\rho>1/2$): Determined by the exit rate β.
  - **Maximal current phase** ($\rho=1/2$): Occurs when both entry and exit rates are high.

2.3.2. Shock Waves and Jamming

- In open systems, a **sharp density jump (shock wave)** can form.
- This is relevant for **traffic congestion** and **ribosome queueing in gene translation**.

2.3.3. Universality and KPZ Scaling

- TASEP belongs to the **Kardar-Parisi-Zhang (KPZ) universality class**.
- It shares statistical properties with **random growth models**.

3. Mathematical Formulation of TASEP

TASEP can be described using **master equations** and **mean-field approximations**.

3.1. Master Equation

For a site i with occupancy n_i, the probability $P(n_1,n_2,\ldots,n_L,t)$ evolves as:

$$\frac{dP}{dt}=\sum_i [P(previous-state)-P(current-state)$$

3.2. Mean-Field Approximation

A simple approximation assumes **independent site occupation probabilities**:

$$\frac{d\rho_i}{dt}=\rho_{i-1}(1-\rho_i)-\rho_i(1-\rho_{i+1})$$

where ρ_i is the probability of site i being occupied.

TASEP is often studied using:

- **Monte Carlo simulations** to track particle movements.
- **Mean-field and hydrodynamic approximations** for analytical insights.
- **Matrix product ansatz** to find exact steady-state distributions.

4. TASEP model discussed by Astik *et al.*

In [4], researchers explore reservoir crowding by studying a Totally Asymmetric Simple Exclusion Process (TASEP) coupled to a finite-resource reservoir. Particle entry and exit rates are dynamically regulated by the reservoir's population, with the total particle number conserved, imposing constraints on system evolution.

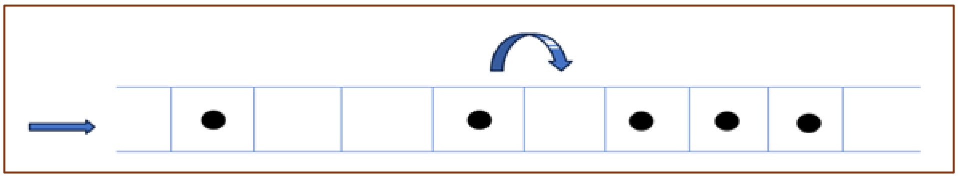


Figure 1: TASEP with open boundary conditions

The model features a TASEP lane (T) connected to a reservoir (R) at both ends. Particles enter T from R, move unidirectionally under the exclusion principle, and eventually return to R, forming a cyclic process. The closed geometry enforces particle conservation (N_0), creating strong correlations between reservoir population and TASEP density, and driving feedback mechanisms that shape steady-state behaviour.

The reservoir is modelled as a point reservoir with unlimited capacity, simplifying the system while retaining key interactions between particle availability and TASEP dynamics.

The TASEP lane, denoted as T, consists of **L discrete sites**, each labelled by an index i such that $i \in [1, L]$. The first site (i=1) represents the **entry point**, where particles from the reservoir R attempt to enter the lane, while the last site (i=L) represents the **exit point**, where particles leave the lane and re-enter the reservoir.

The **entry and exit rates** of the TASEP lane are denoted by **α** and **β**, respectively:

- α represents the **rate at which particles enter** site i=1 from the reservoir R.
- β represents the **rate at which particles exit** site i=L and return to the reservoir.

In this model, the **actual entry and exit rates** of the TASEP lane are not fixed but are **dynamically regulated** by the population of the reservoir R. Specifically, the effective rates are given by:

$$\alpha_e = \alpha f_1(N),\ \beta_e = \beta f_2(N)$$

where:

- N represents the **instantaneous number of particles** in the reservoir R.
- $f_1(N)$ and $f_2(N)$ are **dynamic coupling functions** that determine how the reservoir population influences the entry and exit rates.
- α and β are **intrinsic parameters** as mentioned earlier that set the baseline entry and exit rates, but their actual values at any moment depend on N.

Their study highlights how steady-state behaviour arises from the interplay between entry and exit rates, influenced by reservoir fluctuations. The reservoir modulates these rates through dynamic feedback, impacting phase transitions, density distributions, and transport properties in the TASEP system.

5. Modified One Dimensional TASEP Model with more than one species

In [5-6], authors study a one-dimensional (1D) lattice system of length N as a model for driven diffusion. The lattice contains three objects: positive particles (+), negative particles (-), and holes (0), interacting as follows:

5.1. Movement

- Positive particles move right; negative particles move left.
- Holes remain stationary, allowing particle movement.

5.2. Passing

- Positive and negative particles pass each other without interaction.

5.3. Boundary Conditions

- Positive particles enter from the left and exit at the right.
- Negative particles enter from the right and exit at the left.

5.4. Dynamics & Applications

- Represents nonequilibrium transport seen in traffic flow, biological transport, etc.
- Exhibits phase transitions, shocks, and traffic-like congestion.

5.5. Role of Holes

- Holes enable movement, similar to gaps in traffic.

This model exhibits right-left symmetry: swapping positive and negative particles while reversing space leaves dynamics unchanged. Consequently, under symmetric conditions, both particle currents are equal.

6. Our proposed model

Inspired by the dynamics of totally asymmetric one-dimensional exclusion model consisting of two species of particles [5,6] with open boundary condition, we consider a closed system consisting of a single TASEP with two species of particles connected to a reservoir, reflecting a constraint on the total number of particles. In the open boundary model [4], each side of a 1D lattice of length L may be occupied by either a positive (+) or a negative (-) particle or it may be empty as we mentioned earlier. As usual notation the positive (+) particles move to the right while the negative (-) particles move to the left. Hence positive and negative particles can enter the lattice from the reservoir at the left and right end respectively at a rate α and leave the lattice from right (left) end respectively at a rate β, where rate α and β are constant. The two kinds of particles may pass each other.

In this work because of finite resource, we have taken the effective entry rate and exit rate to be controlled dynamically by the reservoir population [4]. The actual entry and exit rates are given by,

$$\alpha_e^{+} = \alpha f(N_+), \beta_e^{+} = \beta g(N_+)$$

$$\alpha_e^{-} = \alpha f(N_-), \beta_e^{-} = \beta g(N_-)$$

Where, N_+ and N_- are the instantaneous population of the two types of particles in the reservoir R. We define filling factor,

$$\mu_+ = \frac{N_+^0}{L} \text{ and } \mu_- = \frac{N_-^0}{L}$$

for the two types of particles, where, N_+^0 and N_-^0 are total population of the two types of particles. Our model, therefore is a 4-parameter model. The non equilibrium steady states are α, β, μ_+ and μ_-. Both α, β are free parameter with $0 \le \alpha, \beta \le \infty$. Since enhance particle content in R leads to greater in flow of particles in TASEP and hinders outflow of particles from T to R, f(N) and g(N) are monotonically increasing and decreasing function of N. We have chosen

$$f(N) = \frac{N}{N_0} \text{ and } g(N) = 1 - f(N) = 1 - \frac{N}{N_0}$$

Where, N_0 is the total number of particles in the system, i.e. $N_0 = N_+^0 + N_-^0$. In this work we have studied the steady state densities and possible phases for different values of parameters.

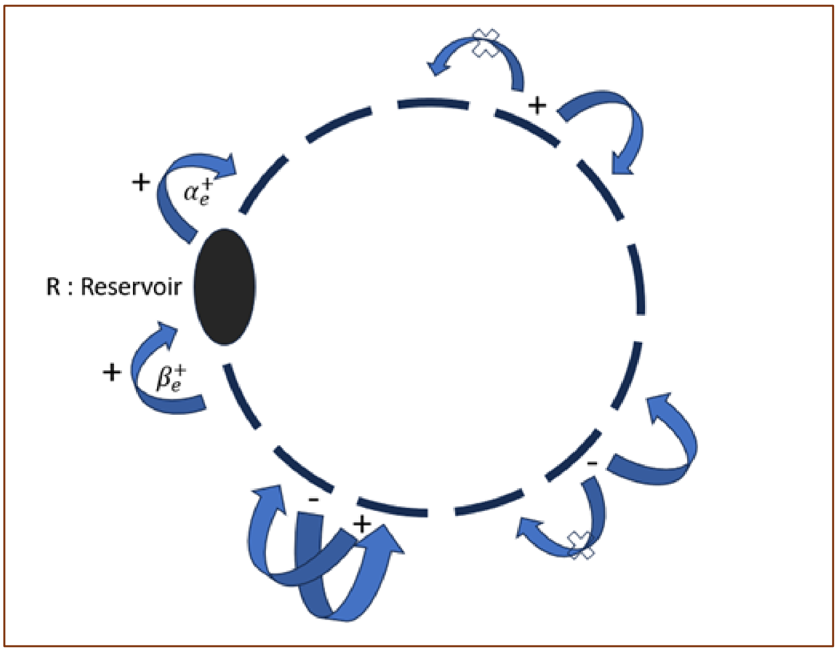
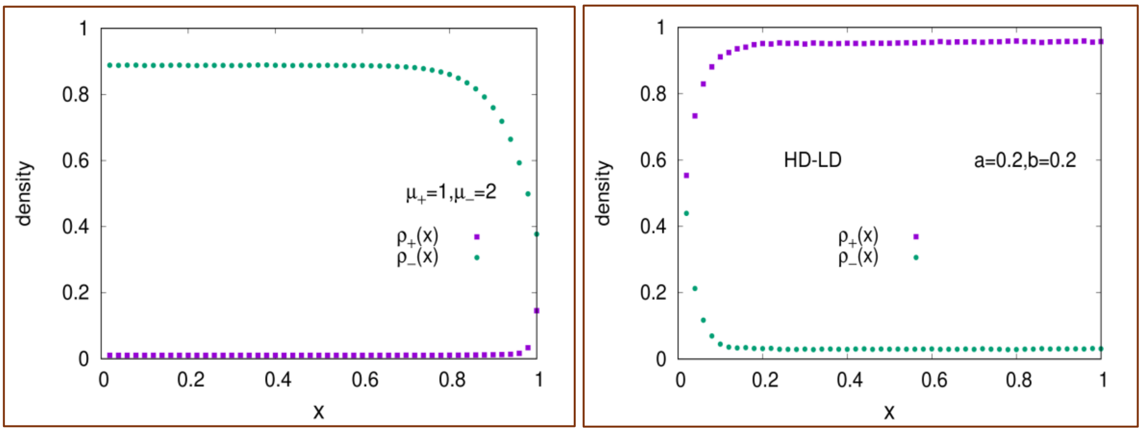


Figure 2: Our proposed TASEP model with two species

7. Results

We use extensive Monte Carlo studies to obtain the phases for various values of the parameter. Since in this model there are two types of particles and no two particles of any type can stay at the same site so HD-HD phases are not possible for any values of μ_+ and μ_-. For very small values of μ_+ and μ_- less than 1/2, there are not enough particles in the system to keep the TASEP lane in its HD or MC phase. So, in this case the only possible phase is the LD-LD phase.

In this write-up we have presented the results for μ_+=50 and μ_-=25 for different values of α and β. We can see HD-LD and MC-MC phases for these parameters. We have also shown the results for α =0.2 and β=0.2 for different values of μ_+ and μ_-. Here we have obtained DW-LD/LD-DW, HD-LD/LD-HD and LD-LD phases.



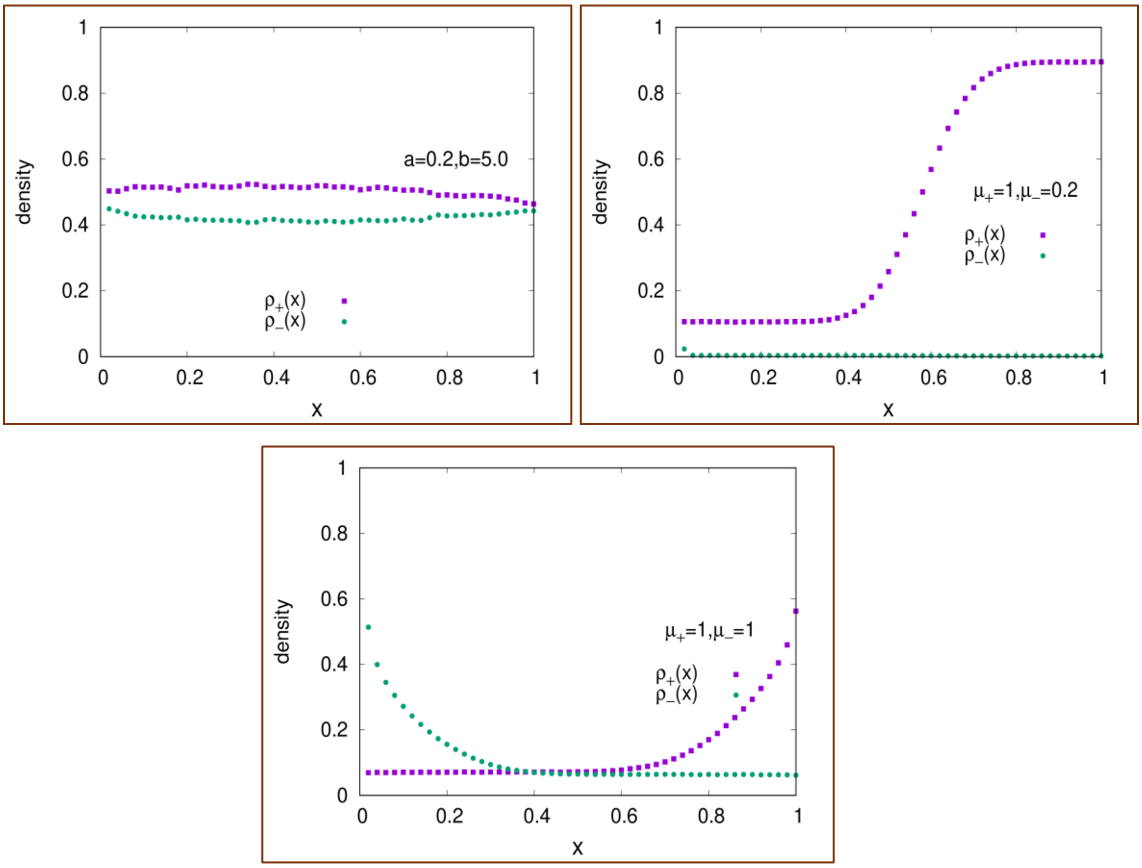


Figure 3: Results of phases for various values of the parameter

8. Conclusion

In this work, we have explored the "crowding effect" of the reservoir on the non-equilibrium states of the TASEP lane with two types of particles, that to our knowledge has previously not been investigated. Here in contrast to the model studied by Evans. Et.al [5-6] the total numbers of both types of particles are constant. Since in the limit of infinite resources the particle number conservation should not be relevant so the large density limit of this model resembles the model studied in Ref. [5-6].

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